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# Determining the two-phase friction coefficient in the transition between annular and fog flow

ENRICO LORENZINI and CARLO CRESCENTINI
Faculty of Engineering, University of Bologna, Italy

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**Abstract**—The dependence of the transition conditions between annular and fog flow on the parameters of a system with boiling flow are analysed in order to extend the applicability of a physical model of the two-phase flow recently proposed by the authors to calculate the friction coefficient. Having suitably represented the phenomenon, examination is made as to its influence on the analytical development of the model, the results obtainable from the latter being compared with those of the relations usually employed and with experimental indications, thus showing the broad field of validity of the formulae presented.

## 1. INTRODUCTION

IN A PREVIOUS paper [1] formulae were deduced, on the basis of a suitable physical model, for calculating the two-phase friction coefficient $\bar{R}$ (local)

$$\bar{R}_l = (dp)_{TP}/(dp)_{LO} \tag{1}$$

where

$(dp)_{TP}$ = pressure drop between two very close sections of a channel traversed by two-phase flow;

$(dp)_{LO}$ = pressure drop between the same two sections when the channel is traversed by a saturated liquid.

As can easily be imagined, the value of this ratio greatly depends on the phenomenology of the two-phase flow; the model suggested differentiates between cases of annular and fog flow. When, however, one is interested in the ratio of the pressure drop not between two infinitely close sections, but between two places at a finite distance from each other, it is necessary to integrate the 'local' expression along the pipe. By far the most interesting case is where the flow is diabatic in one component and the quality and void fraction of the two-phase mixture increase in the direction of motion. The type of flow set up in a section of the channel depends on system pressure, mass velocity and the quality; thus transition may take place between annular and fog motion in the diabatic case for the representation of which it was supposed in ref. [1] that the change in the type of flow happens at a certain void fraction $\alpha_T$, called transition $\alpha$. In practice, having assigned the thermal flow acting on the pipe one can get the void fraction variation with the position, then calculate the friction losses by using different formulae above and below the section where $\alpha = \alpha_T$.

Analytical integration of the expressions obtained is possible only for extremely simple thermal flow distributions, for instance when it is uniform.

In this last case, the two-phase friction multiplier $\bar{R}$ (symbol meanings are shown in the Nomenclature), with $\alpha = 0$ in the initial section of the channel and saturated liquid: for $\alpha_f \leqslant \alpha_T$, $Re_{lo} \gg 2100$, $Re_{gf} \gg 2100$

$$\begin{aligned}\bar{R} = &\frac{1}{x_f}\frac{1}{3T}\left[1-\frac{P^3}{(P+T\alpha_f)^3}\right] \\ &+\frac{P}{x_f}\frac{f_{gf}\rho_l(S-1)^2}{f_{lo}\rho_g}\left\{\frac{-\sqrt{\alpha_f}}{3T(P+T\alpha_f)^3}\right. \\ &+\frac{1}{T^4}\left[\frac{1}{8}\left(\frac{-T}{P}\right)^{5/2}\cdot\frac{1}{2}\ln\frac{|\sqrt{\alpha_f}-\sqrt{(P/-T)}|}{|\sqrt{\alpha_f}+\sqrt{(P/-T)}|}\right. \\ &\left.\left.+\frac{1}{8}\cdot\left(\frac{T}{P}\right)^2\frac{\sqrt{\alpha_f}}{\alpha_f+(P/T)}-\frac{1}{12}\frac{\sqrt{\alpha_f}}{(P/-T)[\alpha_f+(P/T)]^2}\right]\right\}\end{aligned} \tag{2}$$

where, with $P = \rho_l/(\rho_g S)$, one gets

$T = 1-\rho_l/(\rho_g S) = 1-P$ (in almost all cases $T < 0$)

an expression valid for $T < 0$; in ref. [1] the case of $T > 0$ is also dealt with:

for $\alpha_f \leqslant \alpha_T$, $Re_{lo} \lesssim 2100$, $Re_{fg} \gg 2100$

$$\begin{aligned}\bar{R} = &-\frac{P^2}{x_f T^3}\left\{\frac{1}{[(P/T)+1]^3}\left[\ln\frac{|\alpha_f-1|\cdot|P/T|}{|\alpha_f+(P/T)|}\right]\right. \\ &+\frac{1/2}{(P/T)+1}\left[\frac{1}{[\alpha_f+(P/T)]^2}-\frac{1}{(P/T)^2}\right] \\ &\left.+\frac{1}{[(P/T)+1]^2}\left[\frac{1}{\alpha_f+(P/T)}-\frac{1}{(P/T)}\right]\right\} \\ &+\frac{P}{x_f}\cdot\frac{f_{gf}\rho_l(S-1)^2}{f_{lo}\rho_g}\left\{\frac{-\sqrt{\alpha_f}}{3T(P+T\alpha_f)^3}\right. \\ &+\frac{1}{T^4}\left[\frac{1}{8}\left(\frac{-T}{P}\right)^{5/2}\cdot\frac{1}{2}\ln\frac{|\sqrt{\alpha_f}-\sqrt{(P/-T)}|}{\sqrt{\alpha_f}+\sqrt{(P/-T)}}\right. \\ &\left.\left.+\frac{1}{8}\left(\frac{T}{P}\right)^2\frac{\sqrt{\alpha_f}}{\alpha_f+(P/T)}-\frac{1}{12}\frac{\sqrt{\alpha_f}}{(P/-T)[\alpha_f+(P/T)]^2}\right]\right\}\end{aligned} \tag{3}$$

## NOMENCLATURE

$D$ hydraulic diameter of the pipe traversed by two-phase flow or equivalent diameter
$f_{lo}, f_{gf}$ Darcy factors corresponding to $Re_{lo}$ and $Re_{gf}$
$G$ mass flow rate
$p$ pressure of the two-phase mixture
$\bar{R}$ two-phase friction coefficient, ratio of the pressure drop between two sections of a channel traversed by two-phase flow to pressure drop between the same two sections when only saturated liquid flows
$\bar{R}_l$ two-phase friction local coefficient

$$Re_{gf} \begin{cases} Re_{lo}(x_f/\sqrt{\alpha_f})(\mu_l/\mu_g)(S-1) \\ \text{if} \quad \alpha_f < \alpha_T \\ Re_{lo}(x_T/\alpha_T)(\mu_l/\mu_g)(S-1) \\ \text{if} \quad \alpha_f > \alpha_T \end{cases}$$

$Re_{lo}$ Reynolds number of the saturated liquid
$S$ slip, ratio of the gaseous to liquid phase velocity
$T$ temperature of the two-phase mixture
$v_l, v_g$ specific volume of the liquid and the vapour
$x$ two-phase mixture titre
$x_T, x_f$ titre corresponding to $\alpha_T$ and $\alpha_f$, respectively.

Greek symbols
$\alpha$ two-phase mixture void fraction
$\alpha_f$ void fraction in the final section
$\alpha_T$ void fraction on transition between annular and fog flow
$\gamma$ surface liquid tension
$\mu_l, \mu_g$ viscosity of liquid and vapour, respectively
$\rho_l, \rho_g$ density of liquid and vapour, respectively.

(still for $T < 0$);

for $\alpha_f \leqslant \alpha_T$, $Re_{lo} \lesssim 2100$, $Re_{fg} \lesssim 2100$

$$\bar{R} = \frac{-P^2}{x_f T^3} \left\{ \frac{1}{[(P/T)+1]^3} \left[ \ln \frac{|\alpha_f - 1| \cdot |P/T|}{|\alpha_f + (P/T)|} \right] + \frac{1/2}{P/T+1} \left[ \frac{1}{[\alpha_f + (P/T)]^2} - \frac{1}{(P/T)^2} \right] + \frac{1}{[(P/T)+1]^2} \left[ \frac{1}{\alpha_f + (P/T)} - \frac{1}{(P/T)} \right] \right\} + \frac{P f_{gf} \rho_l}{f_{lo} \rho_g} \sqrt{\frac{1}{\alpha_f}} (S-1)^2 \frac{1}{2T^3} \times \left[ \frac{1}{(P/T)^2} - \frac{1}{[\alpha_f + (P/T)]^2} \right] \quad (4)$$

for $\alpha_f \leqslant \alpha_T$, $Re_{lo} \gg 2100$, $Re_{fg} \lesssim 2100$

$$\bar{R} = \frac{1}{x_f} \frac{1}{3T} \left[ 1 - \frac{P^3}{(P + T\alpha_f)^3} \right] + \frac{P f_{gf} \rho_l}{f_{lo} \rho_g} \times \sqrt{\frac{1}{\alpha_f}} (S-1)^2 \frac{1}{2T^3} \left[ \frac{1}{(P/T)^2} - \frac{1}{[\alpha_f + (P/T)]^2} \right] \quad (5)$$

for $\alpha_f > \alpha_T$ in the preceding formulae, always chosen on the basis of $Re_{lo}$ and $Re_{fg}$, but where $\alpha_f$ has been replaced by $\alpha_T$, the following term is added:

$$\left(1 - \frac{x_T}{x_f}\right) \left[ 1 + \frac{\rho_l/\rho_g - 1}{2} (x_f + x_T) \right]$$

(for further details, see [1]).

The $\alpha_T$ parameter plays an important role in determining $\bar{R}$ whose course depends, as the quality in the pipe output section varies, on how the transition parameter changes with system conditions.

From this point precise information is necessary as to the type of two-phase flow changes from which physically significant $\alpha_T$ values can be obtained. Unfortunately, experimental data are scarce and mostly refer to experiments on two-component adiabatic systems.

In ref. [1], since it was desired to check the first reported formulae on flows without excessively variable characteristics, the assumption was made that $\alpha_T$ was constant or, similarly, that only the void fraction determined the type of flow. The results obtained with $\alpha_T = 0.8$—suggested by experimental indications and numerical tests—showed the low sensitivity of $\bar{R}$ to variation of the mass velocity and the need to increase $\alpha_T$ when the pressure is very high compared to that of the flows examined.

It must, on the other hand, be noted that from the conceptual viewpoint, study of the physical models for calculation of the two-phase friction multiplier is separate from that of the transition conditions; however, the formulae previously seen were deduced in attempting to satisfy two requisites: the first is their derivation from a physical model and from analysis of the phenomena and not from numerical artifices on the data, etc., so one must, as far as possible, compare the assumptions made on the course of $\alpha_T$ with the little that is known experimentally; the second is the possibility of using the formulae in the wider field of system functioning, without in any way expecting great precision, but having reliable results on variation of the characteristics of our flow. To achieve this it is absolutely indispensable to know transition condition behaviour.

The need has already been pointed out in ref. [1] for careful study of $\alpha_T$; the aim of this article is to develop this and show the reliability of the formulae proposed.

## 2. TRANSITION BETWEEN ANNULAR AND FOG FLOW

As previously mentioned, the type of flow set up in a pipe—in which a two-phase flow passes, depends—apart from the void fraction—on system pressure and mass velocity; this will be kept in mind by considering $\alpha_T$ a function of $p$ and $G$.

This function structure will be obtained from the laboratory executed observations inherent in flow transitions and the literature reports results of experiments carried out on flows with physical, chemical and geometric characteristics very different from those of interest to us. From these data one can thus obtain the qualitative course of the phenomenon, but certainly not quantitative indications. Therefore, certain numerical assumptions will be provided, justified following a check of the results.

From analysis of the diagrams by Baker and other authors [2–4] or the correlations proposed by Quandt [5], it emerges that, on increasing $p$, $\alpha_T$ must also be increased while, by increasing $G$, $\alpha_T$ decreases. The latter fact is particularly significant because of the previously reported formulae; in fact, if the value of $\alpha_T$ is reduced in them, one gets smaller $\bar{R}$s and the same experimental data course as Muscettola [6] for whom the two-phase friction multiplier is a decreasing function of mass velocity.

Deeper examination then shows that, having fixed a value for pressure, the curve $x_T = f(G)$—where, of course, $x_T$ and $\alpha_T$ are bound by relations

$$\alpha_T = \frac{1}{1+[(1-x_T)/x_T]\psi} \qquad \psi = \frac{\rho_g}{\rho_l} S$$

may be approximated for suitable intervals of the variables (among them are those of technical interest), with a branch of the hyperbola, and that by varying $p$ this hyperbola varies in shape, softening and moving vertically in the ($x_T$–$G$)-plane.

Therefore, we have put

$$x_T = \left(\frac{A_1}{G} + B_1\right)(A_2 + B_2\,e^{-C_2 p}). \tag{6}$$

Among the various structures approximating the dependence of $x_T$ on $p$ the exponential one seemed the most physically opportune, especially due to the monotony of the first and second derivations.

To consider $x_T$ the product of two factors, one dependent on $G$, the other on $p$, implies the supposition that the relationship between such qualities, for $G$ fixed and variable $p$, is independent of the particular value of the mass velocity (a perfectly symmetrical statement when $p$ is fixed and $G$ varied); it has been assumed that such relationships be those that can be obtained from Baker's diagrams for $G \simeq 800$ kg m$^{-2}$ s$^{-1}$ and which should be representative of average values.

On the basis of an interpretation of Muscettola's results on $\bar{R}_1$ (local) the values of $\alpha_T$ for several values of $G$ at 69 bar, may then be fixed. If a greater number of dependable experimental data were available on transitions between flow types, the course of $\alpha_T$ could be studied more accurately, thus removing that last part—an apparent numerical artifice—from the entire reasoning.

It must, however, be noted that the choices made give results that agree with those obtained from the very few experimental data noted in the literature and, above all, it is noted afterwards that the $\alpha_T$ obtained are very similar to those that could have been obtained from Baker's curves, the small differences with the latter being qualitatively explainable by the difference in the working characteristics.

It was, therefore, assumed that (where $p$ is in bar and $G$ is in kg m$^{-2}$ s$^{-1}$):

| | | | | |
|---|---|---|---|---|
| for | $G = 816$ | and | $p = 0.97$ | $x_T = 0.04$ |
| | $G = 816$ | | $p = 55.16$ | $x_T = 0.28$ |
| | $G = 816$ | | $p = 103.43$ | $x_T = 0.38$ |
| | $G = 1000$ | | $p = 68.95$ | $x_T = 0.24$ |
| | $G = 1800$ | | $p = 68.95$ | $x_T = 0.09$, |

Having deduced the values of $A_1, B_1, A_2, B_2$ and $C_2$ one gets:

$$x_T(G, p) = [(1067.622413/G) - 0.308424](0.476615 - 0.442864\,e^{-0.014721 p}). \tag{7}$$

The formula is obviously senseless for $G > 3461$ kg m$^{-2}$ s$^{-1}$ (the second member first factor becomes negative) but it is advisable not to use it for mass velocities exceeding 2000–2500 kg m$^{-2}$ s$^{-1}$, above which there is a zone in Baker's diagram which is difficult to interpret; most applicative cases should, however, be covered. In fact, for very high $G$, supposing one may speak of stabilized fog flow, one would not be far out by assuming $x_T$ as practically zero.

As regards pressure, it is advisable for this not to be much above 137.90 bar; in this case, too, the field of technical applications of two-phase flows is more than amply covered.

The programme for automatic calculation of $\bar{R}$, mentioned in ref. [1], had already been drafted in such a way as to enable the functional dependence of $\alpha_T$ to be easily replaced: it is therefore used also for the checks that will now be discussed.

## 3. APPLICATIONS AND COMPARISONS

For general observations on the formulae proposed, reference should always be made to ref. [1]; the introduction of more precise values for the void fraction on transition serves to overcome the problems set out in the first paragraph, greatly enlarging the field in which the results are reliable besides, as has been seen, solving methodological problems.

$\bar{R}$ comes to be a function in the nature of a refrigerant, final quality, geometry of the system, pressure and velocity of the boiling fluid and the last of these parameters comes into play above all through its influence on $\alpha_T$; it is well to note that the expressions habitually used to calculate that coefficient, keeping account of all these variables, are very few. For example, the classical Martinelli–Nelson diagrams [7], completely neglect the mass velocity and the same occurs for Levy's correlation [8] and all the formulae based on extremely simplified physical models which, generally speaking, are not even able to consider pipe dimensions.

In the two following diagrams the results obtained by the formulae for systems with these characteristics, are graphically reported.

For Fig. 1 (system 1):

—saturated boiling in a pipe whose hydraulic diameter is $D = 0.005$ m;
—liquid at saturation temperature in the pipe input section;
—mass velocity $G = 1000$ kg m$^{-2}$ s$^{-1}$;
—system pressure variable between 45.5 and 85.9 bar.

The different curves refer to different quality values of the mixture in the output sector (to be exact, from the lowest to the highest curve: $x_f = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40$).

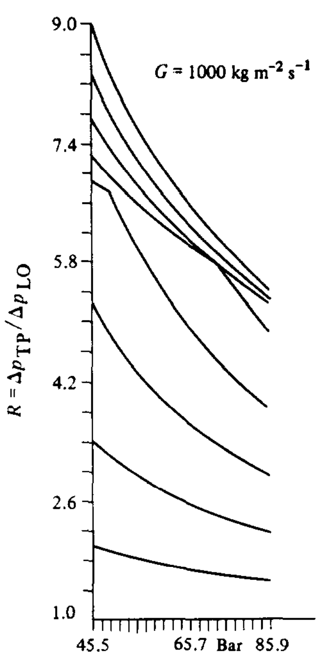


FIG. 1. $R = \Delta p_{tf}/\Delta p_{lo}$ for the system 1 whose characteristics are related in the text; from the lowest to the highest curve: $x_f = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40$.

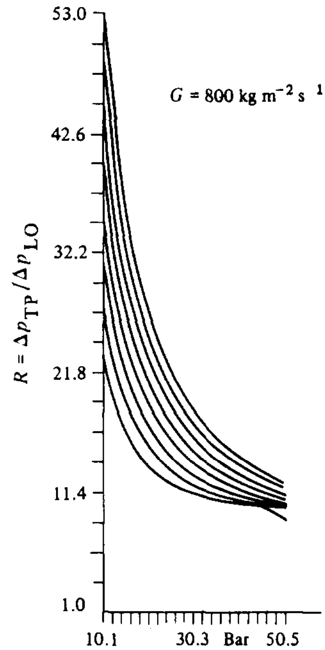


FIG. 2. $R = \Delta p_{tf}/\Delta p_{lo}$ for the system 2 whose characteristics are related in the text; from the lowest to the highest curve: $x_f = 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60$.

For Fig. 2 (system 2):

—saturated boiling in a pipe whose hydraulic diameter is $D = 0.01$ m;
—liquid at saturation temperature in the pipe input section;
—mass velocity $G = 800$ kg m$^{-2}$ s$^{-1}$;
—system pressure variable between 10.1 and 50.5 bar.

The different curves refer to different quality values of the mixture in the output sector (to be exact, from the lowest to the highest curve: $x_f = 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60$).

In Figs. 3–6, instead, a comparison is made for the two previously seen systems between the two-phase friction factor value obtained with the suggested formulae and the one deducible from the Martinelli–Nelson diagram (continuous line: suggested formulae; dashed line: Martinelli–Nelson values).

It may be observed that $\bar{R}$ given by the formulae tends to be lower than Martinelli's,* this is in accordance with available experimental information [2–6] and may be confirmed by the fact that this tendency is more marked for higher pressures. Martinelli and Nelson, in fact, obtained their graphs by basing their results on the integration of correlations valid for flows with a constant quality, due to Martinelli

* This tendency is not general; indeed it is very important that in particular conditions it be inverted, showing considerable agreement with particular experiments as is discussed later.

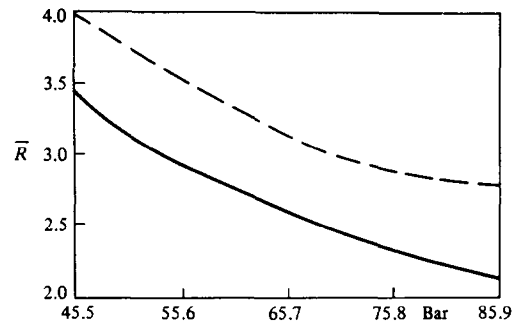


FIG. 3. Comparison between $R = \Delta p_{TP}/\Delta p_{LO}$ obtained with the suggested formulae and with the Martinelli–Nelson diagram for the first on the systems previously analysed with outlet quality: $x_f = 0.10$ (continuous line: suggested formula; dashed line: Martinelli–Nelson values).

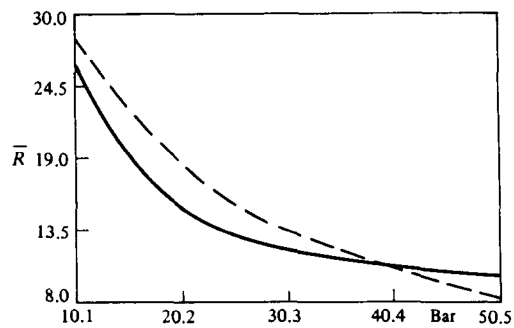


FIG. 5. Comparison for the second system analysed with final quality $x_f = 0.30$; at higher pressures the values obtained with the suggested formulae again become lower than Martinelli and Nelson's and, as in the other graphs, their difference remains almost constant.

himself and to Lockhart; but more often the experiments showed that those correlations overestimate $\bar{R}_l = (dp)_{TP}/(dp)_{LO}$ local for most values of $G$. Again, to explain the difference in the results, it is well to remember that the Martinelli–Nelson diagram, though generalized in use, was obtained by extrapolating data relative to annular flows, a fact which is often forgotten, yet which also explains why $\bar{R}$ is overestimated.

Again, the experiments point out also at high pressure the considerable influence of mass velocity on the two-phase friction factor, which the suggested formulae succeed in reproducing over a wide interval; as already mentioned, the values deduced by Martinelli and Nelson are not dependent on $G$.

Marchaterre's correlation, an improvement on Levy's, gives local $\bar{R}$ as a function of $G$, but greatly underestimated and having a course which, on varying $p$, does not follow experimental results.

Several expressions have been suggested for calculating local $\bar{R}$ which are valid in rather limited system parameter intervals, but deduced from the results of *ad hoc* tests where an attempt was made to bring out the influence of the different variables in play. From the technical viewpoint they are not particularly useful nor even a valid term of comparison because they sometimes contrast with one another. However, some of them, being based on a really large number of experiments, do succeed in having relatively wide applicability and structural simplicity. Among these the Lombardi and Pedrocchi [9] formula has been given particular attention and may serve to compare the suggested formulae with the laboratory results.

According to this formula, the pressure drop—due to friction—along an infinitesimal tract of the channel along which the two-phase mixture passes, is given by:

$$dP_{friction} = kG^n\gamma^{0.4}v_m^{0.86}D^{-1.2}\,dz,$$

where:

$\gamma$ surface liquid tension
$v_m$ average specific volume, that is $xv_g+(1-x)v_l$
$D$ equivalent diameter of the pipe

and, where the sizes are expressed in S.I.

$$k = 0.83$$
$$n = 1.4.$$

In non-adiabatic conditions $x$ varies with $z$ and one can integrate along the channel to obtain $\Delta p_{friction}$ (for the calculations reported below integration was done analytically).

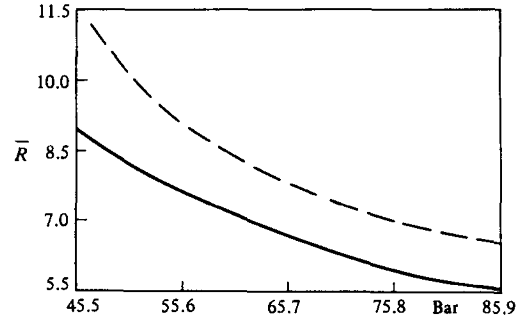


FIG. 4. Comparison for the same system as Fig. 3, but with outlet quality $x_f = 0.40$.

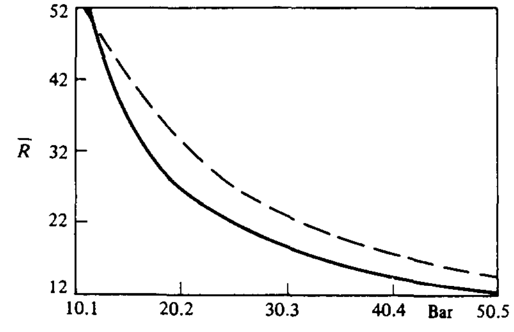


FIG. 6. Comparison for the same system as Fig. 5, but with an outlet quality $x_f = 0.60$.

Table 1. System characteristics: $G = 800\,\mathrm{kg\,m^{-2}\,s^{-1}}$, $p = 10.13$ bar $= 146.96$ psi, $L = 1$ m, $D = 0.01$ m. Direction of motion: vertical; $\gamma = 34.34 \times 10^{-3}\,\mathrm{N\,m^{-1}}$, $v_l = 0.001128\,\mathrm{m^3\,kg^{-1}}$, $v_g - v_l = 0.190791\,\mathrm{m^3\,kg^{-1}}$, $Re_{lo} = 53727.3$, $f = 0.020922$

| $x_f$ | $\Delta p_{friction}$ proposed formulae (Pa × $10^5$) | $\bar{R}$ proposed formulae | $\Delta p_{friction}$ Lombardi–Pedrocchi formulae (Pa × $10^5$) | Observations |
|---|---|---|---|---|
| 0.2 | 0.144 | 19.1 | 0.214 | The $\bar{R}$ are in very good |
| 0.3 | 0.205 | 27.2 | 0.299 | agreement with those |
| 0.4 | 0.268 | 35.4 | 0.379 | obtainable from the |
| 0.5 | 0.330 | 43.8 | 0.457 | Martinelli–Nelson diagram |
| 0.6 | 0.394 | 52.1 | 0.533 | |
| 0.7 | 0.457 | 60.5 | 0.607 | |
| 0.8 | 0.521 | 68.9 | 0.679 | |
| 0.9 | 0.584 | 77.3 | 0.751 | |

The Lombardi–Pedrocchi formula gives $\Delta p_{friction}$ greater than those proposed, so $\bar{R}$ too are superior to Martinelli–Nelson, though one is no longer at low pressure (see discussion of this problem at the end of the paragraph).

In Tables 1–4 this pressure drop is compared with that obtainable from

$$\Delta p_{friction} = \frac{G^2 \cdot L}{2 \cdot \rho \cdot D} f \bar{R}$$

where $f = 0.00560 + 0.5/(GD/\mu)^{0.32}$, valid for $3 \times 10^3 < Re < 3 \times 10^6$] and $\bar{R}$ is given by the suggested formulae.

To obtain $\gamma$ the relation

$$\gamma = \gamma_0[1-(T/T_c)]^n,$$

$$= 75.5 \times 10^{-3}\,\mathrm{N\,m^{-1}}, \quad T_c = 374°\mathrm{C}, \quad n = 1.2$$

was used [10].

Unfortunately, to be valid the Lombardi and Pedrocchi formula must be $0.2 \times 10^{-3}\,\mathrm{N\,m^{-1}} < \gamma < 0.8 \times 10^{-3}\,\mathrm{N\,m^{-1}}$; further comparisons cannot be made at higher pressures, those at which nuclear systems operate and on whose data the suggested formulae have in part been constructed.

In general it must be noted that at 10.13 bar the Lombardi–Pedrocchi formulae give values for the pressure drop which are higher than those deducible from the Martinelli–Nelson equation, contrary to the suggested formulae, which are in good agreement with the two American authors.

When pressure increases, the two formulae continue to improve their agreement and, above all, the percentage variations of $\bar{R}$ become analogous as $G$ increases. On the whole it seems that on varying mass velocity the suggested formulae have a smoother course than that of Lombardi and Pedrocchi (this lesser sensitivity at high pressures is justified by the experimental results but should also be valid at low $p$ and high $G$ considering what is said further on).

If one examines the suggested formula at very low final qualities, lower than the transition quality, one becomes aware of one of its greatest defects: scarce sensitivity to variation of $G$ in these conditions, due to the fact that $G$ is kept in mind, thanks mostly to $x_T$. The consequence of all this is that with very low final qualities* the formula gives an $\bar{R}$ practically independent of $G$, just as if one were using the Martinelli–Nelson correlation; the positive aspect is that this $\bar{R}$ seems to be in better agreement with the experimental than the Martinelli or Lottes–Flinn data and, besides, that $G$ influences the values of $x_f$ for which this inconvenience is verified in such a way that they be very low, unless mass velocity is very small.

The following table illustrates what has been said in the particular case of low pressures (the experimental data are dealt with by Huang and El-Wakil): an atmospheric pressure system is considered with $G = 169.53\,\mathrm{kg\,m^{-2}\,s^{-1}}$ (Table 5) and (also at atmospheric pressure) $G = 135.62\,\mathrm{kg\,m^{-2}\,s^{-1}}$ (Table 6).

Incidentally, it should be noted that Huang and El-Wakil presented their data to show that at low pressure the two-phase friction coefficient obtainable from the Martinelli diagram for qualities of a few per cent is clearly lower than the real one: this tendency is the opposite of that seen at higher pressures, which was the basis for numerical development of the relation between $\alpha_T$, $p$ and $G$. Nevertheless, in those conditions the suggested formulae bring out higher values than Martinelli's and, as we shall see further on, are in good agreement as to size with the laboratory data for all $G$s of practical interest. This may be indirect confirmation of the assumptions made on $\alpha_T$ but above all it shows that the physical model on which the formulae are based, simple though it is, reproduces the salient aspects of two-phase flow. In fact, for final qualities not

* Moreover, in such conditions they become important phenomena which are not considered in the physical model utilized, so use of the suggested formulae must be carefully examined (for further details see the article cited).

Table 2. System characteristics: $G = 1200$ kg m$^{-2}$ s$^{-1}$, $p = 10.13$ bar $= 146.96$ psi, $L = 1$ m, $D = 0.01$ m. Direction of motion: vertical; $\gamma = 34.34 \times 10^{-3}$ N m$^{-1}$, $v_l = 0.001128$ m$^3$ kg$^{-1}$, $v_g - v_l = 0.190791$ m$^3$ kg$^{-1}$; $Re_{lo} = 80596.4$, $f = 0.019057$

| $x_f$ | $\Delta p_{friction}$ proposed formulae (Pa × $10^5$) | $\bar{R}$ proposed formulae | $\Delta p_{friction}$ Lombardi–Pedrocchi formulae (Pa × $10^5$) | Observations |
|---|---|---|---|---|
| 0.2 | 0.277 | 17.9 | 0.378 | The $\bar{R}$ are slightly lower |
| 0.3 | 0.408 | 26.3 | 0.527 | than those of Martinelli– |
| 0.4 | 0.539 | 34.8 | 0.669 | Nelson (the difference |
| 0.5 | 0.670 | 43.3 | 0.806 | diminishes with increasing |
| 0.6 | 0.801 | 51.7 | 0.940 | $x_f$ both in absolute and |
| 0.7 | 0.932 | 60.2 | 1.070 | relative values) |
| 0.8 | 1.062 | 68.6 | 1.198 | |
| 0.9 | 1.193 | 77.1 | 1.324 | |

In this instance the Lombardi–Pedrocchi formula gives values of $\Delta p_{friction}$ superior to those obtainable with Martinelli–Nelson. The $\bar{R}$ obtainable by that formula on increasing $G$ from 800 to 1200 kg m$^{-2}$ s$^{-1}$, are proportionately reduced more than the reported $\bar{R}$.

very low but lower than the transition value, the $\alpha_T$ effect does not come into the agreement at all if not for the fact that, thanks to its structure, it does not intervene.

The experiments then show that by increasing $G$ beyond 169 kg m$^{-2}$ s$^{-1}$ $\bar{R}$ does not vary much and keeps very near to what was calculated with the expressions being examined; by reducing it, however, the two-phase friction factor may become very high and no longer capable of description either by the suggested formulae or the others usually employed. The uncertain data one has, for example, for atmospheric pressure and $G = 88.15$ kg m$^{-2}$ s$^{-1}$ in some cases differ by as much as 100% from the correlations previously examined.

## 4. CONCLUSIONS

Introduction of the functional dependence of $x_T$ from $p$ and $G$ really makes the formulae for calculating $\bar{R}$ valid in a very wide interval of the magnitudes influencing pressure drop due to friction in the two-phase motion.

The comparisons made in the preceding paragraph show that, for $G$ variable from about 100 to 2500 kg m$^{-2}$ s$^{-1}$ and with $p$ between atmospheric pressure and 138 bar or even higher, the suggested expressions are reliable; beyond those limits they still give the correct order of magnitude for the two-phase friction multiplier.

Table 3. System characteristics: $G = 800$ kg m$^{-2}$ s$^{-1}$, $p = 40.5$ bar $= 587.84$ psi, $L = 1$ m, $D = 0.01$ m; direction of motion: vertical; $\gamma = 19.88 \times 10^{-3}$ N m$^{-1}$, $v_l = 0.001254$ m$^3$ kg$^{-1}$, $v_g - v_l = 0.047835$ m$^3$ kg$^{-1}$, $Re_{lo} = 75046$, $f = 0.019368$. The Lombardi–Pedrocchi formula may be used for $\gamma > 20 \times 10^{-3}$ N m$^{-1}$ in this case if it is at the limit of that interval and its results can regularly be removed by more than 15% from the experimental ones

| $x_f$ | $\Delta p_{friction}$ proposed formulae (Pa × $10^5$) | $\bar{R}$ proposed formulae | $\Delta p_{friction}$ (Lombardi–Pedrocchi formulae (Pa × $10^5$) | Observations |
|---|---|---|---|---|
| 0.2 | 0.064 | 8.2 | 0.061 | The $\bar{R}$ are lower than those |
| 0.3 | 0.083 | 10.7 | 0.082 | of Martinelli–Nelson. For |
| 0.4 | 0.090 | 11.6 | 0.101 | final high qualities there is |
| 0.5 | 0.100 | 12.9 | 0.120 | good agreement with |
| 0.6 | 0.112 | 14.4 | 0.138 | Lombardi–Pedrocchi |
| 0.7 | 0.125 | 16.0 | 0.156 | values and those obtainable |
| 0.8 | 0.138 | 17.7 | 0.174 | using Martinelli–Nelson |
| 0.9 | 0.151 | 19.5 | 0.191 | |

Note that the difference between the two $\Delta p_{friction}$ for high $x_f$ is around 20% of the value obtainable with the Lombardi–Pedrocchi formula which, however, was used slightly outside its field of validity, and that in these conditions should overestimate the pressure drop.

Table 4. System characteristics: $G = 1200\,kg\,m^{-2}\,s^{-1}$, $p = 40.5$ bar $= 587.84$ psi, $L = 1$ m, $D = 0.01$ m; direction of motion: vertical; $\gamma = 19.88 \times 10^{-3}\,N\,m^{-1}$, $v_l = 0.001254\,m^3\,kg^{-1}$, $v_g - v_l$ $0.190791\,m^3\,kg^{-1}$, $Re_{lo} = 112570$, $f = 0.017692$. As for the preceding table the value of $\gamma$ is at the limit of the validity interval of the Lombardi and Pedrocchi formula

| $x_f$ | $\Delta p_{friction}$ proposed formulae ($Pa \times 10^5$) | $\bar{R}$ proposed formulae | $\Delta p_{friction}$ Lombardi–Pedrocchi formulae ($Pa \times 10^5$) | Observations |
|---|---|---|---|---|
| 0.2 | 0.093 | 5.8 | 0.108 | $\bar{R}$ are lower than Martinelli–Nelson. $\Delta p_{friction}$ obtained by Lombardi and Pedrocchi is lower than that obtainable with the Martinelli–Nelson $\bar{R}$ |
| 0.3 | 0.118 | 7.4 | 0.144 | |
| 0.4 | 0.146 | 9.1 | 0.178 | |
| 0.5 | 0.175 | 10.9 | 0.211 | |
| 0.6 | 0.204 | 12.8 | 0.244 | |
| 0.7 | 0.234 | 14.6 | 0.275 | |
| 0.8 | 0.264 | 16.5 | 0.306 | |
| 0.9 | 0.294 | 18.4 | 0.337 | |

As for the data of the preceding table, the deviation between $\Delta p_{friction}$ calculated with the two different formulae does not differ too much from the tolerance percentage of the experimental formula, for which the observations already made are valid. Here, however, on increasing the titre the relative deviation diminishes. On the whole, then, it seems that by increasing $G$ the $\bar{R}$ obtainable by Lombardi and Pedrocchi is influenced less than that of the suggested formulae, apart from very high qualities.

Table 5.

| $x_f$ | $R$ (experimental) | $R$ (suggested formula) | $R$ (Martinelli–Nelson) | $R$ (Lottes–Flinn) |
|---|---|---|---|---|
| 0.02 | 33 | 18.5 | ~7 | 19.2 |
| 0.03 | 40 | 35.9 | ~10 | 38.1 |
| 0.04 | 44 | 59.2 | ~14 | 64.0 |
| 0.05 | — | 88.2 | ~19 | 97.2 |

Table 6.

| $x_f$ | $R$ (experimental) | $R$ (suggested formula) | $R$ (Martinelli–Nelson) | $R$ (Lottes–Flinn) |
|---|---|---|---|---|
| 0.02 | 35.6 | in practice values equal to those of Table 5 (discards on the tenths) | identical values to those of the preceding tables | |
| 0.03 | 53.3 | | | |
| 0.04 | 70.0 | | | |
| 0.05 | 83.3 | | | |

For $G$ and $p$ typical of industrial and nuclear installations, and $x_f$ above a few per cent, as in the two cases reported in the diagrams, the experimental data is reproduced with better agreement than all the habitually used correlations in this type of calculation.

As for the problems pointed out at the end of the third section, above all the influence of $G$ on $x_T$ is such that the latter is very low in the technical cases of interest and, in any case, the formulae give good results also for qualities lower than the transition one. Thus the formulae may be used generally even for $x_f$ of the order of the units per cent with an approximation often better than other expressions found in the literature.

The only exception: $\bar{R}$ is clearly underestimated if one simultaneously checks that $x_f < x_T$, the mass flow rate is very small ($G < 100$ kg $m^{-2}\,s^{-1}$) and pressure low (approximately atmospheric).

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## DETERMINATION DU COEFFICIENT DE FROTTEMENT DIPHASIQUE DANS LA TRANSITION ENTRE LES ECOULEMENTS ANNULAIRES ET DE BROUILLARD

**Résumé**—La dépendance des conditions de transition entre écoulements annulaire et de brouillard vis-à-vis des paramètres d'un système d'écoulement avec ébullition, est analysée de façon à élargir l'applicabilité d'un modèle physique d'écoulement diphasique récemment proposé par les auteurs pour calculer le coefficient de frottement. Ayant convenablement représenté le phénomène, on examine l'influence sur le développement analytique, les résultats étant comparés avec ceux des relations habituellement utilisées et aussi avec les données expérimentales, pour montrer le large champ de validité des formules présentées.

## BESTIMMUNG DES REIBUNGSBEIWERTS EINER ZWEIPHASENSTRÖMUNG IM ÜBERGANGSGEBIET ZWISCHEN RING- UND SCHAUMSTRÖMUNG

**Zusammenfassung**—Es wird die Abhängigkeit der Bedingungen für den Übergang zwischen Ring- und Schaumströmung von den Systemparametern bei Strömungssieden untersucht, um die Anwendbarkeit eines physikalischen Modells der Zweiphasenströmung zu erweitern, das von den Autoren zur Berechnung des Reibungsbeiwerts früher vorgeschlagen wurde. Nach ausreichender Darstellung des Phänomens werden die Einflußfaktoren auf die analytische Herleitung des Modells untersucht. Die mit dem Modell ermittelten Ergebnisse werden mit den Werten aus den üblicherweise angewendeten Beziehungen und mit experimentellen Daten verglichen. Es zeigt sich ein weiter Gültigkeitsbereich der vorgeschlagenen Gleichung.

## ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТА ТРЕНИЯ ДЛЯ ДВУХФАЗНОГО ПОТОКА (ТУМАНА) В КОЛЬЦЕВОМ КАНАЛЕ

**Аннотация**—Для расширения диапазона применимости физической модели двухфазного течения, ранее предложенной авторами для расчета коэффициента трения, анализируется зависимость условий переноса в потоке тумана для кольцевого канала от параметров системы. При некоторых упрощениях модели получены результаты, которые сравниваются с обычно применяемыми зависимостями и с экспериментальными данными. Показано, что диапазон справедливости предложенных формул значительно увеличился.